

Gauge-flation: Inflation From Non-Abelian Gauge Fields

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Inflationary models are usually based on dynamics of one or more scalar fields coupled to gravity. In this work we present a new class of inflationary models, *gauge-flation* or non-Abelian gauge field inflation, where slow-roll inflation is driven by a non-Abelian gauge field. This class of models are based on a gauge field theory with a generic non-Abelian gauge group minimally coupled to gravity. We then focus on a particular gauge-flation model by specifying the action for the gauge theory. This model has two parameters which can be determined using the current cosmological data and has the prospect of being tested by Planck satellite data. Moreover, the values of these parameters are within the natural range of parameters in generic grand unified theories of particle physics.

Inflationary Universe paradigm [1, 2], the idea that early Universe has undergone an inflationary (accelerated expansion) phase, has appeared very successful in reproducing the current cosmological data through the Λ CDM model [1, 2]. Many models of inflation have been proposed and studied so far, e.g. see [3], which are all compatible with the current data. Inflationary models are generically single or multi scalar field theories with standard or non-standard kinetic terms and a potential term, which are minimally or non-minimally coupled to gravity. Generically, in these models inflationary period is driven by a “slowly rolling” scalar field (inflaton field) whose kinetic energy remains small compared to the potential terms.

Toward the end of inflation the kinetic term becomes comparable to the potential energy, and inflaton field(s) start a (fast) oscillation around the minimum of their potential losing their energy to other fields present in the theory, the (p)reheating period. The energy of the inflaton field(s) should eventually be transferred to standard model particles, reheating, where standard FRW cosmologies take over. Therefore, to have a successful cosmology model one should embed the model into particle physics models. With the current data the scale of inflation (or Hubble parameter H during inflation) is not restricted well enough, it can range from 10^{14} GeV to the Big Bang Nucleosynthesis scale 1 MeV. However, larger H , $H \gtrsim 10$ GeV, is preferred within the slow-roll inflationary models with preliminary particle or high energy physics considerations. It is hence natural to tune the inflationary model within the existing particle physics models suitable for similar energy scales.

Most of successful inflationary scenarios so far use scalar field(s) as the inflaton, because turning on time dependent scalar fields does not spoil the homogeneity and isotropy of the cosmology. Although it is relatively easy to write down a potential respecting the slow-roll dynamics conditions, it is generically not easy to argue for such potentials and their stability against quantum corrections within particle physics models. For example, the Higgs sector in the ordinary electroweak standard model minimally coupled to Einstein gravity does not support

a successful inflationary model e.g. see [4]. The situation within beyond standard model theories seems not to be better.

Vector gauge fields are commonplace in all particle physics models. However, their naive usage in constructing inflationary models is in clash with the homogeneity and isotropy of the background. It has been argued that this obstacle may be overcome by introducing many vector fields which contribute to the inflation, such that the anisotropy induced by them all average out [5]. Alternatively one may introduce three orthogonal vector fields and retain rotational invariance by identifying each of these fields with a specific direction in space [5]. Nonetheless, it was shown that it is not possible to get a successful vector inflation model in a *gauge invariant* setting [5]. Lack of gauge invariance, once quantum fluctuations are considered may lead to instability of the background and may eventually invalidate the background classical inflationary dynamics analysis [6].

Here, we construct a new class of vector inflation models and to avoid the above mentioned possible instability issue we work in the framework of gauge field theories. In addition, to remove the incompatibility with isotropy resulting from gauge fields we introduce three gauge fields. We choose these gauge fields to rotate among each other by $SU(2)$ non-Abelian *gauge* transformations. Explicitly, the rotational symmetry in 3d space is retained because it is identified with the global part of the $SU(2)$ gauge symmetry. In our model we need not restrict ourselves to $SU(2)$ gauge theory and, since any non-Abelian gauge group has an $SU(2)$ subgroup, our *gauge-flation* (non-Abelian gauge field inflation) model can be embedded in non-Abelian gauge theories with arbitrary gauge group. Another advantage of using non-Abelian gauge theories is that, due to the structure of non-Abelian gauge field strength, there is always a potential induced for the combination of the gauge field components which effectively plays the role of the inflaton field.

In the above discussions we have only committed ourselves to the gauge invariance and have not fixed a specific gauge theory action. This action will be fixed on the requirement of having a successful inflationary model.

We study one such gauge-flation model but gauge-flation models are expected not to be limited to this specific choice. In this Letter we consider a simple two parameter gauge-flation model and study classical inflationary trajectory for this model as well as the cosmic perturbation theory around the inflationary path. We then use the current data for constraining the parameters of our model and show that our model is compatible with the current data within a natural range for its parameters.

The inflationary setup. Consider a 4-dimensional $su(2)$ gauge field A_μ^a , where a, b, \dots and μ, ν, \dots are respectively used for the indices of the gauge algebra and the space-time. We will be interested in *gauge invariant* Lagrangians $\mathcal{L}(F_{\mu\nu}^a, g_{\mu\nu})$ which are constructed out of metric $g_{\mu\nu}$ and the strength field F

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g\epsilon_{bc}^a A_\mu^b A_\nu^c, \quad (1)$$

where ϵ_{abc} is the totally antisymmetric tensor. We work with FRW inflationary background metric

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j, \quad (2)$$

where indices i, j, \dots label the spatial directions.

The effective inflaton field is introduced as follows: We will work in temporal gauge $A_0^a = 0$ and at the background level, as in any inflationary model, we only allow for t dependent field configurations [7]

$$A_\mu^a = \begin{cases} \phi(t)\delta_i^a, & \mu = i \\ 0, & \mu = 0. \end{cases} \quad (3)$$

With this choice we are actually identifying our gauge indices with the spatial indices. That is, we identify the rotation group $SO(3)$ with the global part of the gauge group, $SU(2)$. Therefore, the rotational non-invariance resulted from turning on space components of a vector is compensated by (the global part of) the gauge symmetry. $\phi(t)$ is not a genuine scalar, while

$$\psi(t) = \frac{\phi(t)}{a(t)} \quad (4)$$

is indeed a scalar. (Note that for the flat FRW metric $e_i^a = a(t)\delta_i^a$, where e_i^a are the 3d triads.) The components of the field strengths in the ansatz are

$$F_{0i}^a = \dot{\phi}\delta_i^a, \quad F_{ij}^a = -g\phi^2\epsilon_{ij}^a. \quad (5)$$

After fixing the gauge and choosing A_0^a to be zero, system has nine other degrees of freedom, A_i^a . However, in the ansatz (3) we only keep one scalar degree of freedom. We should hence first discuss consistency of the reduction ansatz (3) with the classical dynamics of the system induced by $\mathcal{L}(F_{\mu\nu}^a, g_{\mu\nu})$. It is straightforward to show that the gauge field equations of motion $D_\mu \frac{\partial \mathcal{L}}{\partial F_{\mu\nu}^a} = 0$, where D_μ is the gauge covariant derivative, i) allows for a solution of the form (3) and, ii) once evaluated on the

ansatz (3) becomes equivalent to the equation of motion obtained from the “reduced Lagrangian” $\mathcal{L}_{red}(\dot{\phi}, \phi; a(t))$,

$$\frac{d}{a^3 dt} (a^3 \frac{\partial \mathcal{L}_{red}}{\partial \dot{\phi}}) - \frac{\partial \mathcal{L}_{red}}{\partial \phi} = 0, \quad (6)$$

where \mathcal{L}_{red} is obtained from inserting (5) and metric (2) into the original gauge theory Lagrangian \mathcal{L} . Moreover, one can show that the energy momentum tensor, $T_{\mu\nu}$, computed over the FRW background (2) and the gauge field ansatz (3) takes the form of a homogeneous perfect fluid

$$T_\nu^\mu = \text{diag}(-\rho, P, P, P),$$

which is the same as the energy momentum tensor obtained from the reduced Lagrangian \mathcal{L}_{red} . That is,

$$\rho = \frac{\partial \mathcal{L}_{red}}{\partial \dot{\phi}} \dot{\phi} - \mathcal{L}_{red}, \quad P = \frac{\partial (a^3 \mathcal{L}_{red})}{\partial a^3}. \quad (7)$$

All the above is true for any gauge invariant Lagrangian $\mathcal{L} = \mathcal{L}(F_{\mu\nu}^a; g_{\mu\nu})$. To have a successful inflationary model, however, we should now choose appropriate form of \mathcal{L} . The first obvious choice is Yang-Mills action minimally coupled to Einstein gravity. This will not lead to an inflating system with $\rho + 3P < 0$, because as a result of scaling invariance of Yang-Mills action one immediately obtains $P = \rho/3$ and that $\rho \geq 0$. So, we need to consider modifications to Yang-Mills. As will become clear momentarily one such appropriate choice is

$$S = \int d^4x \sqrt{-g} \left[-\frac{R}{2} - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \frac{\kappa}{384} (\epsilon^{\mu\nu\lambda\sigma} F_{\mu\nu}^a F_{\lambda\sigma}^a)^2 \right] \quad (8)$$

where we have set $8\pi G \equiv M_{\text{pl}}^{-2} = 1$ and $\epsilon^{\mu\nu\lambda\sigma}$ is the totally antisymmetric tensor. This specific F^4 term is chosen because the contribution of this term to the energy momentum tensor will have the equation of state $P = -\rho$, making it perfect for driving inflationary dynamics. (To respect the weak energy condition for the F^4 term, we choose κ to be positive.) The reduced (effective) Lagrangian is obtained from evaluating (8) for the ansatz (3):

$$\mathcal{L}_{red} = \frac{3}{2} \left(\frac{\dot{\phi}^2}{a^2} - \frac{g^2 \phi^4}{a^4} + \kappa \frac{g^2 \phi^4 \dot{\phi}^2}{a^6} \right). \quad (9)$$

Energy density ρ and pressure P are then given by

$$\rho = \rho_{YM} + \rho_\kappa, \quad P = \frac{1}{3} \rho_{YM} - \rho_\kappa, \quad (10)$$

where

$$\rho_{YM} = \frac{3}{2} \left(\frac{\dot{\phi}^2}{a^2} + \frac{g^2 \phi^4}{a^4} \right), \quad \rho_\kappa = \frac{3}{2} \kappa \frac{g^2 \phi^4 \dot{\phi}^2}{a^6}. \quad (11)$$

Recalling the Friedmann equations

$$\begin{aligned} H^2 &= \frac{1}{2} \left(\frac{\dot{\phi}^2}{a^2} + \frac{g^2 \phi^4}{a^4} + \kappa \frac{g^2 \phi^4 \dot{\phi}^2}{a^6} \right), \\ \dot{H} &= - \left(\frac{\dot{\phi}^2}{a^2} + \frac{g^2 \phi^4}{a^4} \right), \end{aligned} \quad (12)$$

the slow-roll parameter ϵ is

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{2\rho_{YM}}{\rho_{YM} + \rho_\kappa}. \quad (13)$$

To obtain a slow-roll inflationary phase initial conditions and parameter κ should be chosen such that ρ_κ dominates over ρ_{YM} during inflation. As slow-roll inflation progresses the contribution of Yang-Mills term ρ_{YM} to the energy momentum tensor grows and eventually at around $\rho_{YM} = \rho_\kappa$ inflation ends. To have a consistent slow-roll inflation, it is not sufficient to have small ϵ ; for any physical quantity X , $\frac{\dot{X}}{HX}$ should remain small. In particular, demanding the effective scalar inflaton field ψ to be slowly varying, i.e. $\delta \equiv -\frac{\dot{\psi}}{H\psi} \ll 1$ and $\dot{\delta}/(H\delta) \ll 1$, yields

$$\epsilon \simeq \psi^2(1 + \gamma), \quad \eta \simeq \psi^2 \quad (14a)$$

$$\delta \simeq \frac{\gamma}{6(\gamma + 1)}\epsilon^2, \quad \kappa \simeq \frac{(2 - \epsilon)(1 + \gamma)^3}{g^2\epsilon^3}, \quad (14b)$$

in the leading order in ϵ . In the above

$$\gamma = \frac{g^2\psi^2}{H^2} \quad \text{or equivalently} \quad H^2 \simeq \frac{g^2\epsilon}{\gamma(\gamma + 1)}, \quad (15)$$

γ is a slowly varying positive parameter of order one. Since $\delta \sim \epsilon^2$ (cf. (14b)), ψ is varying slower than ϵ and hence from (15) we learn that during slow-roll regime

$$\frac{\epsilon}{\epsilon_i} \simeq \frac{\gamma + 1}{\gamma_i + 1}, \quad \frac{\gamma}{\gamma_i} \simeq \frac{H_i^2}{H^2}, \quad (16)$$

where ϵ_i , γ_i and H_i are the values of these parameters at the beginning of inflation. Number of e-folds N_e at the end of inflation, marked by $\epsilon_f = 1$, is then given by

$$N_e = \int_{t_i}^{t_f} H dt = - \int_{H_i}^{H_f} \frac{dH}{\epsilon H} \simeq \frac{\gamma_i + 1}{2\epsilon_i} \ln \frac{\gamma_i + 1}{\gamma_i}. \quad (17)$$

The value of ψ at the beginning and end of inflation are related as $\psi_f^6 \simeq \frac{1}{2}\psi_i^6$, where (14b) has been used and by \simeq sign we mean equality to the leading order in slow-roll parameter ϵ . Notice that all the dimensionful quantities, like κ , ψ and H , are measured in units of M_{pl} .

Gauge-flation cosmic perturbation theory. So far we have analyzed dynamics of the homogeneous effective scalar inflaton field ψ , while consistently turning off the other gauge field components. To compare our model with the data we should work out the power spectrum of curvature perturbations and their spectral tilt for which we need to study cosmic perturbation theory in gauge-flation. In general small fluctuation around the ansatz (3) can be parameterized by 12 fields δA_μ^a . Decomposing μ index into time and spatial parts and identifying the gauge index a with the spatial index i , these 12 fields give

rise to four scalars, three divergence-free vectors and a divergence-free, traceless symmetric tensor:

$$\delta A_0^a = \delta^{ak} \partial_k \dot{Y} + \delta_j^a u^j,$$

$$\delta A_i^a = \delta_i^a Q + \delta^{aj} \partial_{ij} (M + \partial_i v_j + t_{ij}) + \epsilon_i^a{}^j (g\phi \partial_j P + w_j),$$

where ∂_i denotes partial derivative respect to x^i , the scalars are parameterized by Y, Q, M, P , vectors by u_i, v_i, w_i and the tensor by t_{ij} . As we see, we are indeed dealing with a multi-field inflationary model. Among the scalars, Q can be identified with the fluctuation of the inflaton field ϕ .

The other field active during inflation is metric whose fluctuations are customarily parameterized by four scalars, two divergence-free vectors and one tensor:

$$ds^2 = -(1 + 2A)dt^2 + 2a(\partial_i B - S_i)dx^i dt + a^2((1 - 2C)\delta_{ij} + 2\partial_{ij}E + 2\partial_{(i}W_{j)} + h_{ij})dx^i dx^j.$$

In the first order perturbation theory which we are interested in, scalar, vector and tensor fluctuations do not couple to each other. Among 12 gauge field perturbations and 10 metric perturbations one scalar and one vector mode of the gauge field, and two scalars and one vector of the metric modes are gauge degrees of freedom. We hence remain with five gauge-invariant scalar, three massless vector and two massless tensor modes.

Equations of motion for the perturbations can be obtained from perturbed Einstein equations $\delta G_{\mu\nu} = \delta T_{\mu\nu}$, which decomposes into four equations for scalar modes, two for vector modes and one equation for tensor modes [2]. The equation of motion for the remaining scalar, vector and tensor mode t_{ij} is provided through perturbed gauge field equations.

A thorough analysis reveals that amplitude of vector perturbations are exponentially suppressed as in the ordinary scalar-driven inflationary models, and that power spectrum of the tensor mode coming from the gauge field t_{ij} , is suppressed at the super-horizon scales, hence it does not contribute to the tensor power spectrum [8].

The full analysis of cosmic perturbation theory in our model has many new and novel features compared to the standard scalar-driven inflationary models, a detailed analysis of which is presented in [8], in the following table we summarize the results:

Power spectrum of curvature perturbations	$\mathcal{P}_{\mathcal{R}}$	$\frac{1}{8\pi^2\epsilon} \left(\frac{H}{M_{\text{pl}}}\right)^2$
Spectral Tilt	$n_s - 1$	$-2(\epsilon - \eta)$
Tensor to Scalar ratio	r	16ϵ
Power spectrum of anisotropic inertia $a^2\pi^S$	$\mathcal{P}_{a^2\pi^S}$	$\frac{\epsilon}{8\pi^2} \left(\frac{H}{M_{\text{pl}}}\right)^2$

A specific feature of gauge-flation is that it predicts a non-zero power spectrum for the scalar anisotropic inertia $a^2\pi^S$ [2], with the ratio

$$\frac{\mathcal{P}_{a^2\pi^S}}{\mathcal{P}_\mathcal{R}} = \epsilon^2 = \left(\frac{r}{16}\right)^2. \quad (18)$$

Note that $a^2\pi^S$ is identically zero in all the scalar-driven inflationary models in the context of Einstein GR.

Confronting gauge-flation with the data. To this end, we depict the results of our model on the allowed region of the $n_s - r$ graph:

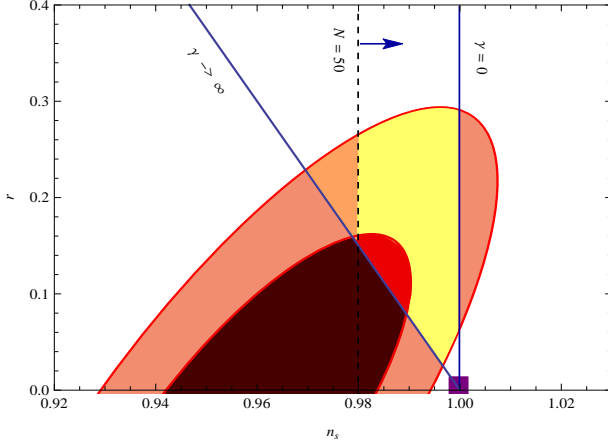


FIG. 1. The 1σ and 2σ contour bounds of 7-year WMAP+BAO+H0 [9]. The blue lines represent the gauge-flation predictions for $\gamma = 0$ and $\gamma \rightarrow \infty$ limits and the region with enough number of e-fold is on the right-side of the $N_e = 50$ line. Therefore, the allowed region is the highlighted region between $N_e = 50$ and $\gamma = 0$ lines. Although the spectral tilt is always red in our model, we have the lower bound $n_s > 0.98$. The minimum value of tensor-to-scalar ratio r , on 2σ contour is 0.03, while on the 1σ contour it is 0.08.

Restricting ourselves to 1σ contour in Fig 1, we find stringent bounds on r , $n_\mathcal{R}$, $\frac{\mathcal{P}_{a^2\pi^S}}{\mathcal{P}_\mathcal{R}}$ and H

$$0.08 < r < 0.15, \quad H \simeq (3.1 - 4.4) \times 10^{-5} M_{\text{pl}}, \quad (19)$$

$$0.98 \leq n_s \leq 0.99, \quad \frac{\mathcal{P}_{a^2\pi^S}}{\mathcal{P}_\mathcal{R}} \simeq (2.5 - 10) \times 10^{-5}, \quad (20)$$

while within the 1σ contour, we have $\gamma > 2$. For $\gamma \sim 10$ which is a natural value for γ , we find the following values for the parameters g and κ

$$\frac{g^2}{4\pi} \sim 10^{-6}, \quad \kappa \simeq 10^{15} M_{\text{pl}}^{-4}, \quad (21)$$

and the gauge field value during inflation turns out to be sub-Planckian $\psi \simeq (2 - 4) \times 10^{-2} M_{\text{pl}}$.

Discussion. We showed that non-Abelian gauge field driven inflation, *gauge-flation*, can lead to a successful slow-roll inflation model with specific features. In

the model we considered the theory has two parameters, gauge coupling g and the coefficient of the $(F\tilde{F})^2$ term κ . The value for the gauge coupling g required by the CMB data is of order 10^{-3} , while Λ , the scale associated with $\kappa \sim \Lambda^{-4}$, is of order 10^{14} GeV . These two parameters are in the natural range for perturbative beyond standard models of particle physics. Moreover, the κ -term may be obtained by integrating out axionic fields where Λ is associated with scale of the axion potential [10, 11]. For this procedure to be theoretically meaningful we need $\Lambda \gg H$, which is respected by the best-fit values of our model.

Current data tightly restricts the values of our parameters. In particular, noting Fig. 1, our model predicts that the tensor-to-scalar ratio r is restricted to be in $0.03 < r < 0.15$ range, which is well within the range to be probed by the Planck satellite. As another prediction, while gauge-flation has always a red spectral tilt, the tilt has a lower bound $n_s > 0.98$.

Finally we point out a specific feature of our model not shared by usual scalar-driven inflationary models: gauge-flation predicts a non-zero scalar anisotropic inertia ($a^2\pi^S \neq 0$), and $\frac{\mathcal{P}_{a^2\pi^S}}{\mathcal{P}_\mathcal{R}} \sim 10^{-4}$. It would be interesting to explore observational prospects this ratio, which we postpone to future works.

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